

# Decision-Support System for the Optimal Technology Split of a Decarbonized Bus Network

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**Abstract**—In recent years, an increasing number of cities has started to deploy electric buses in test or demonstration phases. Among many different variants of electric technologies, usually a small number of buses of a selected technology type is ordered and operated on a single bus line. Few experiences were made with regard to a full fleet conversion, which requires an in-depth assessment of local operating conditions for different technologies and is accompanied by a number of complex, interrelated strategic and operational decisions. The goal of this work is to identify an optimal composition of electric technologies for the fleet of an urban bus network. Specifically, hydrogen-powered fuel cell buses, overnight or opportunity charging buses can be chosen. By minimizing total cost of ownership, the optimal technology for each bus line is selected and vehicle and charging schedules as well as spatial distribution of charging stations and infrastructure dimensions at the depot are optimized.

## I. INTRODUCTION

The EU's commitment to achieve a significant reduction of carbon emissions has generated considerable interest in using electric buses for public transportation. Electric vehicles use electric motors instead of internal combustion engines for propulsion and do not have any tail pipe emissions. Energy can be provided in different ways. Besides the commonly known concept of using batteries as energy storage systems, other technologies, such as supercapacitors or fuel cells, which use refuelled hydrogen and generate electricity on board, are available on the market. Each of these technology options has its distinct pros and cons with respect to driving ranges, filling and charging requirements or cost. The urban context of a bus network adds an individual component to the optimal technology choice. A transportation networks' bus schedules and routes may facilitate one, but complicate the deployment of other technology concepts, since they determine energy consumption, circulation and turning times as well as the potential location of charging stations. In many cases, individual bus lines have characteristics which make it most suitable for one technology, while other bus lines fit better for other concepts. Therefore, a technology mix, namely a fixed, but maybe different technology decision for each bus line, can present the most cost-efficient solution for a given bus network.

The following technology concepts are considered as viable options in our project:

### A. Fuel Cell

Fuel Cell (FC) buses use electrical energy generated on board through an electrochemical reaction of hydrogen and air. As these propulsion systems offer high flexibility in terms of range, operational deployment similar to that of conventional Diesel buses is feasible. However, the daily consumption of energy requires a nearby hydrogen filling station and regular supply of hydrogen. Supply can be realized through an off-site production plant and subsequent delivery via trucks or an on-site hydrogen production plant, depending on the number of fuel cell buses deployed. In any case, the storage of large quantities of hydrogen at the filling station requires compliance with the Seveso-III-Directive (2012/18/EU). Therefore, it will be preferred to position a number of smaller dimensioned filling stations at several locations.

### B. Opportunity Charging

The opportunity charging (OPC) concept is based on the idea of frequently recharging buses during dwell times. Thus, a number of charging stations has to be located at suitable bus stops along the routes. By scheduling charging events of more than one bus line at charging stations at shared bus stops, synergies from joint usage of infrastructure can be gained and peaks in power demand can be reduced. As this concept is based on using high charging power levels, the use of fast charging batteries is required. These batteries are more expensive than traditional automotive batteries, but allow a greater number of loading cycles under high power levels and increase the batteries life expectancy. Since they usually have smaller capacities, buses highly depend on regular recharges and the operational flexibility of bus operators is reduced. On the other side, high charging power levels permit shorter charging activities during the day, which has a positive effect on the availability and required number of buses.

### C. Overnight Charging

The overnight charging (ONC) concept assumes that charging mainly takes place during night, when buses are not in operation. A major benefit of using these long available timeslots at the depot is the possibility of deploying low charging power levels. For large-scale deployment it has to be considered that simultaneous charging of the fleet poses high

requirements on the electricity grid. If existing infrastructure cannot provide enough capacity, the necessary upgrade of the electrical infrastructure results in high investment cost. Moreover, range limitations of overnight charging buses make it usually impossible to directly replace one conventional powered bus by one electric bus. In many cases, buses must recharge their batteries after several hours of operation. The reduced availability of electric buses during these charging activities is further strengthened by arising deadhead trips from and to the depot. Therefore, the utilization of charging stations at centrally located company-owned properties can be incorporated in vehicle schedules.

The optimal technology choice among these options highly depends on urban context and topology and can differ between bus lines. In order to determine the optimal mix for any given bus network, specifically, an optimal technology decision for each line, we developed a Mixed Integer Linear Program (MILP). The present paper describes the key components of this model and is organized as follows. Section II gives a brief literature overview of related studies. In Section III, a general problem description is presented. Section IV contains the formal representation of the model. Solution method, preliminary results and directions for future work are discussed in Section V.

## II. LITERATURE

Early studies dealing with related problems can be found as the vehicle scheduling problem with time constraints (VSPT). The VSPT considers the standard VSP and involves restrictions on the maximum time a vehicle may spend away from depot. [1] and [2] were among the first papers to investigate efficient methods for solving this kind of problem. In an attempt to provide more problem specific solutions, [3] refined the VSPT by adding charging time constraints. As the relevant aspect of limited driving range is travelled distance, the VSP with route distance constraints (VSP-RC), as described in [4], is generally more applicable to represent practical problems. [5] studied this problem for different types of electric and non-electric buses. As the VSP-RC does not allow to exploit the very specific problem structure of the VSPT, the large number of resulting route constraints was handled with a column-generation-based algorithm. Another relevant column generation approach was suggested in [6], where two models for scheduling depot charging buses were proposed.

The optimal location or restricted capacity of charging stations was not considered in any of these described models. [7] addressed this problem and proposed a heuristic framework to optimize depot charging buses of two different types in consideration of their required infrastructure. Based on this description of a network flow problem, a linear model was developed by [8]. Apart from the novelty of a linear problem formulation, also charging stations at selected terminal stops and partial charging were considered.

Besides the presented selection of studies dealing with technologies here described as ONC, also OPC received increasing attention. A detailed introduction into this topic is given in [10]. In this work, the joint optimization of charging infrastructure and battery capacity was stressed and the recharge process and other technical aspects were covered in-depth. Nevertheless, the overlapping of charging events at a station only received limited attention. In contrast, [9] proposed a model that ensured that not more than one electric bus can be recharged on each individual charger at any time. The output included decisions about the location of charging stations, the number of needed chargers and the corresponding recharging schedule. Also [11] presented a model for optimizing the distribution of charging infrastructure and focused on synergies from using mixed fleets. Besides opportunity charging, also biofuels were considered as potential technology options. The introduced optimization model gave valuable insights into the advantages of mixed fleets and outlined the importance of the urban context of a bus network.

The present work contributes to the literature by considering the optimal technology split among a distinct set of technologies and by considering collectively used charging stations, which requires a special focus on operational aspects.

## III. PROBLEM DESCRIPTION

The primary objective of the developed model is to identify the optimal technology  $q \in \{FC, OPC, ONC\}$  for each bus line  $l$ . As this decision affects strategic and operational planning, the different levels of managerial decisions are combined by minimizing Total Cost of Ownership (TCO), which include all expenditures arising throughout the life cycle of a product or service. In order to represent TCO of an electric bus network, the general cost drivers *distance*, *vehicle*, *depot infrastructure* and *network infrastructure* were identified. The corresponding cost parameters serve as cost centres and account for all related expenditures within the planning horizon.  $cost^{distance}$  are defined for individual route sections within the bus network and consist of running energy cost. Route sections are defined as scheduled trips from one terminal to another or deadhead trips, which connect final stops of different lines or final stops and depot. Since buses of different technologies rely on different strategies to ensure energy supply, route deviations and vehicle schedules may vary between technologies. For the same reason, the number of necessary buses to determine total *vehicle* related cost may differ between technologies. Cost arising from *depot infrastructure* can be classified into three categories: charging station, hydrogen filling station and grid connection cost, each of which taking the form of a step function. The cost for charging stations at the depot  $cost^{\beta}$  are defined by the number of overnight charging buses. Depending on the bus type, each vehicle receives a designated charger and based on the number of chargers at a charging station, a certain number of buses increases cost by a constant value. The cost of the hydrogen filling station  $cost^{H2}$  are a function of total daily hydrogen demand and required safety stock. Depending

on the corresponding number of electrolyzers, compressors and coolers used per filling station, a specific amount of cost is added. The cost for the required grid connection at the depot  $cost^{kW}$  are subject to the combined power demand of charging and hydrogen infrastructure at a location. As the existing medium-voltage power grid is insufficient for providing several additional megawatts of connection power, the enlargement of the grid requires high investments, when certain limits are exceeded. Therefore, depot cost for different scales of connection power are also taken into account by a step function. Finally, cost for *network infrastructure* refer to cost that arise from distributing charging stations at bus stops or company-owned properties within the network. This final cost center consists of  $cost^{station}$ , which represent initial land preparation and infrastructure cost and  $cost^{charger}$ , which arise for each additional charger installed at a charging station.

The developed model can be structured into several parts: the base model and technology-specific sub models. The base model represents above described cost structures and ensures, that each bus line is assigned to a certain technology. Information about the required number of vehicles  $x_q^\beta$ , the necessary grid capacity at the depot  $x_q^{kW}$  and the total daily hydrogen demand  $x_q^{H2}$  of each technology  $q$  is received from vehicle schedules, which are derived in the respective sub models. In order to determine vehicle schedules, a network based on a directed graph  $G = (V, A)$  with  $V = V^t \cup V^c \cup d^{out} \cup d^{in}$  and  $A = \{(s, t), \dots\}$  is defined for each possible bus technology.  $V^t$  is the set of trip nodes, which have to be serviced on a working day. Each trip  $t$  is defined by a concrete start and end time and location and belongs to a specific line  $l$ . The set of nodes  $V^c$  consists of optional charging events  $c$ ,  $d^{out}$  and  $d^{in}$  are source and sink nodes at the depot, respectively. An arc  $(s, t)$  connects node  $s$  with node  $t$  and can represent deadhead or waiting arcs for a certain technology. Arcs between trip nodes are only included, when the subsequent execution of trips is temporally feasible. Moreover, an upper time limit on waiting times at bus stations is imposed to reduce the number of trip connections. In general, the problem consists of finding paths through the directed network, such that a number of technology-specific side conditions are respected and total incurred cost are optimized.

#### A. Fuel Cell

The FC network does not rely on external charging during operations, therefore the set of charging events  $V^c$  is empty. A vehicle schedule is given by a sequence of nodes, starting and ending at the depot. Each travelled arc and trip node in the network is characterized by a specific consumption value, the sum of these gives the daily hydrogen demand.

#### B. Opportunity Charging

In order to ensure sufficient battery levels for opportunity charging buses, a total charge time of  $charge\_time_l$  has to be scheduled per rotation. As there can be several potential charging stations along a line, charging can be spread across

different stations as needed. A solution in terms of timing and length of charging events at a station of a line is repeated throughout the day. To model these regular events in the OPC network, optional charging nodes are inserted after trip nodes terminating at suitable bus stops. A charging event  $c$  is characterized by a specific location  $n$ , an earliest possible charging start time  $start_c$  and a maximum charging duration  $charge\_time_l$ . Given this maximum charging duration and some additional maximum waiting time, the set of potential successive trip nodes is reduced. Moreover, it is assumed that trip nodes following a charging event must start at the same location. When scheduling start and end times of charging events, attention must be paid to the overlapping charger use of buses of different lines.

#### C. Overnight Charging

In the ONC network, buses have large battery capacities, which are assumed to be fully charged, when vehicles start their daily operations. In contrast to OPC, the occurrence of charging events in vehicle schedules and their respective total length per rotation is not specified in advance. Buses with low mileage may not require additional charge, others have to perform recharging activities and must be replaced by extra vehicles during that time. Recharging takes place at strategically located sites in the network, such as the depot or other company-owned properties. As the need for charging events is not known beforehand, deadhead trips and optional charging nodes are inserted at all terminal stops of bus lines. The start of charging activities is assumed to take place immediately after arrival at the charging site and charging station occupation is optimized through the flexibility of leaving regular vehicle schedules at any trip node. Though full charges are not necessarily required during the day, also charging events of very short lengths are rather unlikely, since route deviations reduce the availability of buses and involve additional cost. In order to set also end times of charging events in advance, the duration of charging events is discretised and only charging events with full and half charge are considered.

### IV. MATHEMATICAL FORMULATION

In this section the mathematical formulation of the above described problem is presented. In table 1, 2 and 3, an alphabetical summary of all introduced sets, variables and parameters is given. For ease of understanding, parameters are uniformly represented in complete words, sets in capital letters and variables and indices in small letters.

The objective function and constraints (2) and (3) are part of the developed base model. The ultimate goal is to minimize TCO of the electric bus network, as stated in (1). Furthermore, information about the required number of vehicles  $x_q^\beta$ , the necessary grid capacity at the depot  $x_q^{kW}$  and the total daily hydrogen demand  $x_q^{H2}$  of each technology  $q$  is processed. The  $x$  variables of each class  $\beta$ ,  $H2$  and  $kW$  are passed to constraint (2) to derive binary variables  $z_i^*$ , which identify the associated segment of the step function.  $step_i^*$  describes the

TABLE I  
INTRODUCED SETS, VARIABLES & PARAMETERS

Sets	
$I_q$	set of cost intervals $i$ for technology option $q$
$L$	set of lines $l$
$Q$	set of technology options $q$
$A$	set of arcs from source $s$ to target $t$
$A_n \subseteq A$	set of incoming arcs of simultaneous charging events at station $n$
$A^-(v)$	set of preceding nodes of node $v$
$A^+(v)$	set of successive nodes of node $v$
$N$	set of potential charging stations $n$
$N_l \subseteq N$	set of potential charging stations $n$ of line $l$
$M$	set of discrete time steps $m$
$V^c$	set of charging nodes $c$
$V^t$	set of trip nodes $t$
Variables	
$a_{(s,t)} \in \{0, 1\}$	1 if arc from source $s$ to target $t$ is used, 0 otherwise
$b_{(l,n)} \in \mathbb{N}$	charging time at charging station $n$ at line $l$
$b_c^{end} \in \mathbb{N}$	end of charging at charging node $c$
$b_c^{start} \in \mathbb{N}$	start of charging at charging node $c$
$\varepsilon_v^+ \in \mathbb{N}$	remaining charge when leaving node $v$
$\varepsilon_v \in \{0, 1\}$	1 if remaining charge at node $v$ forbids charging, 0 otherwise
$f_{(c,m)}^{end*} \in \{0, 1\}$	1 if charging event $c$ did not end by time step $m$ , 0 otherwise
$g_{(c,m)}^{start} \in \{0, 1\}$	1 if charging event $c$ already started at time step $m$ , 0 otherwise
$t_{(q,l)} \in \{0, 1\}$	if technology $q$ is chosen for line $l$ , 0 otherwise
$u_{(c,m)} \in \{0, 1\}$	1 if charging event $c$ is taking place in time step $m$ , 0 otherwise
$v_n \in \mathbb{N}$	number of necessary chargers at station $n$
$w_n \in \{0, 1\}$	1 if station $n$ is built, 0 otherwise
$x_q^\beta \in \mathbb{N}$	number of buses of technology $q$
$x_q^{H2} \in \mathbb{N}$	hydrogen demand of technology $q$
$x_q^{kW} \in \mathbb{N}$	power demand of technology $q$
$x^* \in \mathbb{N}$	argument of step function: $* \in \{\beta, H2, kW\}$
$z_i^* \in \{0, 1\}$	1 if cost step $i$ of cost function $*$ is chosen, 0 otherwise
Parameters	
$charge_c$	charge amount at charging event $c$
$chargers_n^{max}$	maximum number of chargers at charging station $n$
$chargetime_c$	charging duration of charging event $c$
$chargetime_l$	necessary charging duration per round of line $l$
$cons_t$	consumption of trip $t$
$cons_{(s,t)}$	consumption from source node $s$ to target node $t$
$cons_l^{kg}$	hydrogen consumption of trips of line $l$
$cons_{(s,t)}^{kg}$	hydrogen consumption of deadhead arc from $s$ to $t$
$cost_q^{bus}$	cost per bus of technology $q$
$cost_{(q,l)}^{consum}$	distance cost per line $l$ operated by technology $q$
$cost_{(s,t)}^{consum}$	distance cost of deadhead arc from $s$ to $t$
$cost_{(q,n)}^{station}$	charging station cost for technology $q$ at station $n$
$cost_{(q,n)}^{charger}$	charger cost for technology $q$ at station $n$
$cost_e^E$	cost of cost interval $e$ for electricity infrastructure
$cost_e^{H2}$	cost of cost interval $e$ for hydrogen infrastructure
$cost_e^Q$	cost of cost interval $e$ for charging infrastructure
$M$	Big-M
$power^{charger}$	charging power for depot charge
$SoC_v^{charge}$	state of charge at node $v$ that allows recharging
$SoC_v^{max}$	maximum state of charge at node $v$
$SoC_v^{min}$	minimum state of charge at node $v$
$start_c$	start of charging event $c$
$start_t$	start of trip $t$
$step_i^*$	step $i$ of step-fixed cost function $*$

level of each cost segment  $i$ , whereas  $*$  is a placeholder for  $\beta$  of each technology  $q$  and the aggregated variables of each class  $kW$  and  $H2$ . Besides the mapping to specific cost intervals, the base model ensures that each bus line  $l$  is assigned to a certain technology  $q$ , which is represented in constraint (3).

$$\min \sum_{q \in Q} \sum_{l \in L} t_{(q,l)} * cost_{(q,l)}^{distance} + \sum_{(s,t) \in A} a_{(s,t)} * cost_{(s,t)}^{distance} + \sum_{i \in I^{kW}} z_i^{kW} * cost_i^{kW} + \sum_{i \in I^{H2}} z_i^{H2} * cost_i^{H2} + \sum_{q \in Q} \sum_{i \in I_q^\beta} z_i^\beta * cost_i^\beta + \sum_{q \in Q} x_q^\beta * cost_q^{bus} + \sum_{n \in N} w_n * cost_n^{station} + v_n * cost_n^{charger} \quad (1)$$

$$\sum_{i \in I} z_i^* * step_i^* = x^* \quad \forall i \in I^* \quad (2)$$

$$\sum_{q \in Q} t_{(q,l)} = 1 \quad \forall l \in L \quad (3)$$

The technology-specific sub models are composed of general constraints (4) - (6), which are applicable across all technologies, as well as some technology-specific constraints. Constraint (4) ensures that if technology  $q$  is chosen for line  $l$ , the sum of incoming arcs of each trip  $t$  of this line is set to 1. Constraint (5) is a flow conservation constraint and guarantees that the the number of incoming arcs equals the number of outgoing arcs at each node. Constraint (6) derives the number of necessary buses to serve all trips by summing up all depot-leaving arcs. A distinction between 12- and 18-meter buses can be considered if the assignment of service trip to bus type is provided in advance.

$$\sum_{v \in A^-(t)} a_{(v,t)} = t_{(q,l)} \quad \forall q \in Q, l \in L, t \in V_l \quad (4)$$

$$\sum_{s \in A^-(v)} a_{(s,v)} = \sum_{t \in A^+(v)} a_{(v,t)} \quad \forall v \in V \quad (5)$$

$$x_q^\beta = \sum_{t \in A^+(d^{out})} a_{(d^{out},t)} \quad \forall q \in Q \quad (6)$$

These general relations are formulated for each individual technology network and extended by technology-specific constraints.

#### A. Fuel Cell

In constraint (7) the amount of daily required hydrogen for infrastructure dimensioning of the FC network is calculated by summing up the hydrogen consumption of fixed lines and deadhead trips. The implications on the required infrastructure at the depot are represented through the variables  $x_q^\beta$ ,  $x_q^{kW}$  and  $x_q^{H2}$  and processed in the base model.

$$x_q^{H2} = \sum_{l \in L} t_{(q,l)} * cons_l^{kg} + \sum_{(s,t) \in A} a_{(s,t)} * cons_{(s,t)}^{kg} \quad (7)$$

$q = \text{FC}$

### B. Opportunity Charging

In order to ensure sufficient battery levels for OPC buses, the total duration of charging events at different stops  $n$  along a line must repeatedly satisfy an upfront calculated charging duration. In constraint (8) the sum of all charging times is calculated for each line  $l$  and fixed to the parameter  $charge_{time}_l$ . Constraint (9) ensures that a positive charging duration  $b_{(l,n)}$  is associated with an activated incoming arc for the respective charging event. Constraint (10) guarantees that only service trips with start times larger than the end time of preceding charging events are selected as successive nodes, whereas constraints (11) and (12) determine start and end time of the charging events. As the optimized use of charging infrastructure can require to wait for chargers to become available, the earliest possible start time of a charging event  $b_c^{start}$  can be shifted by  $s_{(l,n)}$  minutes. The possibility to spread recharging activities of different lengths among several potential charging locations can be seen as a further instrument to minimize simultaneous charger occupation. Since start time and duration of charging events are not known beforehand, the overlapping of charging events is addressed in equations (13) to (17). In constraints (13) and (14) the auxiliary variables  $g_{(c,m)}^{start}$  and  $f_{(c,m)}^{end}$  are determined. The binary variables  $u_{(c,m)}$  to mark the occupation of charging event  $c$  in time step  $m$  are set in (15). In constraint (16) the maximum number of simultaneously used chargers is calculated. Next, an upper bound on the number of chargers is imposed for each potential charging location  $n$  in constraint (17). Finally, variable  $w_n$  indicates the utilization of charging location  $n$  and is used to represent initial land preparation cost.

$$\sum_{n \in N_l} b_{(l,n)} = charge_{time}_l \quad \forall l \in L \quad (8)$$

$$a_{(t,c)} * charge_{time}_l \geq b_{(l,n)} \quad \forall c \in V^c, t = A^-(c), l = line_c, n = location_c \quad (9)$$

$$b_c^{end} \leq start_t * a_{(c,t)} + (1 - a_{(c,t)}) * M \quad \forall c \in V^c, t \in A^+(c) \quad (10)$$

$$b_c^{start} = a_{(t,c)} * start_c + s_{(l,n)} \quad \forall c \in V^c, t = A^-(c), l = line_c, n = location_c \quad (11)$$

$$b_c^{end} = b_c^{start} + b_{(l,n)} \quad \forall c \in V^c, l = line_c, n = location_c \quad (12)$$

$$b_c^{start} \geq (1 - g_{(c,m)}^{start}) * (m + 1) \quad \forall c \in V^c, m \in M \quad (13)$$

$$m \geq b_c^{end} - M * f_{(c,m)}^{end} \quad \forall c \in V^c, m \in M \quad (14)$$

$$u_{(c,m)} = f_{(c,m)}^{end} + g_{(c,m)}^{start} - 1 \quad \forall c \in V^c, m \in M \quad (15)$$

$$v_n \geq \sum_{c \in V_n} u_{(c,m)} \quad \forall n \in N, m \in M \quad (16)$$

$$v_n \leq chargers_n^{max} \quad \forall n \in N \quad (17)$$

$$w_n * chargers_n^{max} \geq v_n \quad \forall n \in N \quad (18)$$

### C. Overnight Charging

In order to ensure that buses operate within predefined battery levels, the current state of charge  $\varepsilon_v^+$  is calculated at each network node  $v$ . At the depot node,  $\varepsilon_v^+$  is set to its maximum value. In constraint (20), a minimum SoC is required for all trip and charging nodes. Constraint (21) and (22) are imposed to transfer battery levels of preceding nodes to trip or charging nodes, respectively. For trip nodes, the state of charge  $\varepsilon_t^+$  when leaving the node  $t$  must be lower than the state of charge  $\varepsilon_v^+$  of the preceding node  $v$  minus the consumption value  $cons_{(v,t)}$  of the deadhead arc  $(v,t)$  and the consumption  $cons_t$  of the service trip  $t$  itself. For charging nodes, the subtracted trip consumption is replaced by the amount of energy  $charge_v$  that is charged during charging event  $v$ . An exact computation of current battery states by providing upper bounds for  $\varepsilon_t^+$  is not performed. Instead, a certain discharge level  $SoC^{discharge}$  must be reached to allow the scheduling of additional charging events during the day, as imposed in constraints (23) and (24). Finally, constraint (25) ensures that the number of buses that simultaneously use charging events at a shared charging station  $n$  satisfy capacity restrictions. As the timing of potential charging events is already known in advance, the use of variables that indicate charger occupation per discrete time step is not required.

$$\varepsilon_{Depot}^+ = SoC_v^{max} \quad (19)$$

$$\varepsilon_v^+ \geq SoC_v^{min} \quad \forall v \in V_t \cup d^{in} \quad (20)$$

$$\varepsilon_t^+ \leq \varepsilon_v^+ - a_{(v,t)} * (cons_{(v,t)} + cons_t) + (1 - a_{(v,t)}) * SoC^{max} \quad \forall t \in V^t, v \in A^-(t) \quad (21)$$

$$\varepsilon_c^+ \leq \varepsilon_s^+ - a_{(s,v)} * (cons_{(s,v)} - charge_v) + (1 - a_{(s,v)}) * SoC^{max} \quad \forall c \in V^c, s \in A^-(v) \quad (22)$$

$$SoC_v^{max} * \varepsilon_t \geq \varepsilon_t^+ - SoC^{discharge} \quad \forall t \in V^t \quad (23)$$

$$a_{(t,c)} \leq 1 - \varepsilon_t \quad \forall t \in V^t, c \in A^+(t) \quad (24)$$

$$\sum_{(t,c) \in A_n} a_{(t,c)} \leq chargers_n^{max} \quad \forall n \in N \quad (25)$$

## V. RESULTS AND DISCUSSION

The presented model was developed to optimize the technology split of the urban bus system in Graz, Austria, a city with roughly 300.000 inhabitants and 80.000 incoming commuters on a regular working day. The network is composed of 4.000 trips, which can be assigned to 34 different bus lines and are serviced by a minimum number of 153 buses, not including replacement vehicles or safety stock. The proposed MILP was implemented in Python and solved with the general purpose solver Gurobi 8.0. As the overall model is fairly complicated and requires considerable running time, it may help to add valid inequalities as described in [12]. This idea will be elaborated in future.

TABLE II  
RESULTS FOR A PRELIMINARY DATA SET.

	FC	OPC	ONC
# of lines	0	14	20
# of buses	0	76	108
# of extra stations	0	4	0
# of extra chargers	0	10	0
# of charging events per day	0	840	28
total charge minutes per day	0	3.295	4.424
total kilometers per day	0	12.360	17.485

As anticipated, our first results show that the optimal solution of the investigated bus network is represented by a technology mix. 14 bus lines are optimally suited for OPC (see table II). In order to ensure periodic recharge activities for these lines at bus stops, a total of 10 chargers will be established at 4 different charging stations within the network, as indicated in figure 1. The required number of buses to operate this part of the network is 76. The other 20 bus lines are serviced by 108 overnight charging buses, some of which requiring additional daytime charging at the depot. Though also company-owned properties in more central locations were considered as potential recharge locations, charging events were solely planned at the depot, where charging stations are available for free. In our test scenario no FC buses were chosen, which can be explained by the high cost of hydrogen and the costly filling infrastructure. These cost values may well decrease as H2 technology evolves and thus may lead to a future inclusion in the technology mix. The full output of the model includes vehicle and charging schedules, besides the necessary fleet size and infrastructure dimensions. A detailed description of provided output and underlying assumptions will be provided in future work.

As the uncertainty of input parameters is a major challenge in this field of application, the presented framework should be understood as a starting point to generate different input scenarios and study the sensibility of initial solutions. As further part of the project, the robustness of the resulting solutions will be tested by means of a simulation model. The results provide insights in complex systematic relationships and can serve as decision support for bus operators and local authorities. Trade-offs between the number of buses, battery

capacities and charging power levels or investment and running cost can be elaborated in detail and help decision-makers as early as in the process of defining tendering specifications.

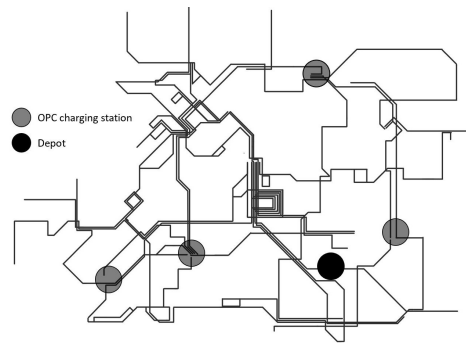


Fig. 1. Selected charging stations within the network.

Our preliminary results suggest that the deployment of a mixed fleet, even though not intuitive, can indeed lead to monetary advantages. A range of other available electric bus technologies, such as battery buses which use fuel cells as range extender or battery buses with In-Motion-Charging are not considered yet. As the used input parameters are still at a preliminary level, more solid input data has to be gathered and further analyses have to be performed. Another research direction will be the incorporation of global emissions in the optimal technology decision, as the current version of the model only focuses on the prevention of local emissions.

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